



University of the Philippines Baguio
College of Science
Department of Mathematics and Computer Science

COURSE SYLLABUS

Math 232
Real Analysis



A. COURSE DETAILS

| | | |
|---------------------|---|-------------|
| Course Number: | Math 232 | |
| Course Name: | Real Analysis | |
| Course Description: | Rigorous introduction to classical real analysis. Brief review of real numbers and a discussion of the topology of metric spaces; includes detailed discussion of the following topics: the analysis of sequences and series; continuity, differentiation and Taylor's theorem; Riemann and Lebesgue integration; measure theory. | |
| Credit Units: | 3 u | |
| Prerequisite: | COI (Consent of Instructor) | |
| Requirements: | Oral Presentations | 10% |
| | Problem Sets | 40% |
| | Midterm Examination | 25% |
| | Final Examination | 25% |
| | Total | 100% |
| Passing Grade: | 60% | |

B. PROGRAM LEARNING OUTCOMES (PLO)

Upon completion of the MS Mathematics program, the students are able to:

- PLO 1** Discuss advanced concepts from various fields of mathematics including, but not limited to Algebra, Analysis, and Statistics, to exhibit mastery in the different areas of mathematics.
- PLO 2** Provide accurate and rigorous solutions and proofs, both verbal and written, for mathematical problems, to demonstrate deep understanding of the theory and its applications.
- PLO 3** Generate new results to contribute to mathematical knowledge.
- PLO 4** Prepare for doctoral studies, advanced research, and careers in education, industry, and government, towards the students' professional development and personal growth.

C. COURSE LEARNING OUTCOMES (CLO)

At the end of this course, the students must be able to:

- CLO 1** Examine the connections between rings, algebras, Dynkin systems, and monotone classes generated by a collection of sets.
- CLO 2** Demonstrate knowledge on the construction, examples, and properties of abstract and concrete measures.
- CLO 3** Define the Lebesgue integral of simple functions, nonnegative measurable functions, Lebesgue integrable functions.
- CLO 4** Discuss the classical convergence theorems of Lebesgue integration and their applications to iterated integrals, transformation rules, boundary integration, and integral inequalities.
- CLO 5** Investigate the various decompositions of signed and complex measures.



D. MAPPING OF CLO with the PLO

| | PLO 1 | PLO 2 | PLO 3 | PLO 4 |
|--------------|-------|-------|-------|-------|
| CLO 1 | F | F | I | M |
| CLO 2 | F | F | I | M |
| CLO 3 | F | F | I | M |
| CLO 4 | F | F | I | M |
| CLO 5 | F | F | I | M |

LEGEND: I-Introduced; M-Moderately achieved; F-Fully achieved

Introduced [30%, 60%]: The course has introduced fundamental concepts necessary for the program objectives.

Moderately Achieved [60%; 85%]: The course has developed partial knowledge, somehow prerequisite, for the program objectives.

Fully Achieved [85%, 100%]: The course has developed complete knowledge associated with the program objectives.

E. CLASS RULES

1. Students are expected to have a copy of any of the reference materials. Lectures and exercises are based on these materials.
2. The University rule on class attendance (Article 346 of the University Code) shall be strictly enforced in the class.
3. If a student misses a short quiz, his/her grade on that quiz is zero. If a student misses a long examination for a valid reason (this requires documentation), his/her grade in the final exam will also account for his/her grade for the missed exam. This applies to no more than one long exam missed. A student who fails to take any examination for invalid reasons will get a grade of 0% for that exam.
4. Cheating, in any form, will not be tolerated.

F. GRADING SCHEME

The work of students shall be graded at the end of each semester/midyear term in accordance with the following system:

| Grade | Interval | Classification: |
|-------|-----------|-----------------|
| 1.00 | [96, 100] | Excellent |
| 1.25 | [91, 96) | |
| 1.50 | [87, 91) | Very Good |
| 1.75 | [82, 87) | |
| 2.00 | [78, 82) | Good |
| 2.25 | [73, 78) | |
| 2.50 | [69, 73) | Satisfactory |
| 2.75 | [64, 69) | |
| 3.00 | [60, 64) | Pass |
| 4.00 | [55, 60) | Conditional |
| 5.00 | [0, 55) | Fail |
| INC | | Incomplete |
| DRP | | Dropped |

Remark: Clearing of Grades of "Incomplete" or "4"

"4" means conditional failure. It may be made up for by successful repetition of the course, or by passing a removal examination. Students are given a grade of "3" if they pass the re-examination, "5" if they fail. Only one



re-examination which must be taken within the prescribed time is allowed. If a student does not remove the grade “4” within the prescribed time, he/she may earn credit for the course only by repeating it and passing it.

The grade of “INC” is given if a student whose class-standing throughout the semester is “Passing,” fails to take the final examination or fails to complete other requirements for the course due to illness or other valid reasons. In case the class standing is not passing and the student fails to take the final examination for any reason, a grade of “5” is given. Removal of the “INC” must be done within the prescribed time by passing an examination or meeting all the requirements for the course, after which the student shall be given a final grade based on his/her overall performance.

Taken from: [UPB Academic Catalogue 2019](#)

| G. COURSE OUTLINE | | | | |
|---------------------------|---|--|---|---|
| Timeline | CLOs | Topics | Learning Activities | Assessment Tools |
| MEASURE THEORY | | | | |
| Week 1 | CLO 1 | 1. Collection of Sets Rings, Algebras, Borel σ -algebras | Lectures; Group Discussions; Reading Assignments; Individual Inquiry; Consultations; Pause for Reflection; Collaborative Work; Class Reporting | Problem Sets; Oral Presentations and Reporting; Written Examination |
| Week 2 | | Semirings, Dynkin Systems, Monotone Classes | | |
| Week 3 | CLO 2 | 2. Measure Theory: Construction, Completion, and Example Contents, Premeasures, Measures | | |
| Week 4 | | From Premeasures to Measures, Lebesgue Measure, Outer and Inner Measures | | |
| Week 5 | | Complete Measure Spaces, Borel and Lebesgue-Stieltjes Spaces, Regular Borel Measures on Metric Spaces | | |
| Week 6 | | Image Measures, Properties of Lebesgue Measure | | |
| INTEGRATION THEORY | | | | |
| Week 7 | CLO 3 CLO 4 | 3. Lebesgue Integration Theory and Convergence Theorems Measurable Real-Valued Functions, Lebesgue Integral of Simple Functions | Lectures; Group Discussions; Reading Assignments; Individual Inquiry; Consultations; Pause for Reflection; Collaborative Work; Class Reporting | Problem Sets; Oral Presentations and Reporting; Written Examination |
| Week 8 | | Lebesgue Integral of Nonnegative Measurable Functions, Lebesgue Integrable Functions | | |
| Week 9 | | Almost Everywhere Properties, Convergence Theorems, Riemann and Lebesgue Integrals | | |
| Week 10 | CLO 2 CLO 4 | 4. Product Measures, Iterated Integrals, and Change of Variables Formula Initial, Final, and Product σ -algebras | | |
| Week 11 | | Product Measures, Fubini–Tonelli Theorem | | |
| Week 12 | | Integration Through Image Measures, Change of Variables for Integration | | |
| Week 13 | | 5. Boundary Integration, Lebesgue Spaces, and Integral Inequalities Boundary Integrals, Gauss Divergence Theorem, Reynolds Transport Theorem | | |
| Week 14 | Lebesgue Spaces, Integral Inequalities: Jensen, Hölder, Minkowski, Lyapunov | | | |
| Week 15 | CLO 5 | 6. Decomposition of Measures Signed and Complex Measures | | |
| Week 16 | | Radon–Nikodym Theorem, Lebesgue and Hahn Decompositions | | |



H. REFERENCES

1. R. Bartle, *The Elements of Integration and Lebesgue Measure*, John Wiley and Sons, New York, 1995.
2. H. Bauer, *Measure and Integration Theory*, Walter de Gruyter, Berlin, 2001.
3. D. Cohn, *Measure Theory*, Birkhäuser, Boston, 1980.
4. J. Elstrodt, *Maß- und Integrationstheorie*, Springer, New York, 2018.
5. T. Hawkins, *Lebesgue's Theory of Integration: Its Origins and Development*, AMS Chelsea Publishing, Providence, 1975.
6. H. Royden, *Real Analysis*, 3rd ed., Macmillan Publishing Company, New York, 1988.
7. W. Rudin, *Real and Complex Analysis*, 3rd ed., McGraw-Hill, New Year, 1979.
8. H. Sohrab, *Basic Real Analysis*, Birkhäuser, Boston, 2003.
9. E. Stein and R. Shakarchi, *Real Analysis : Measure Theory, Integration, and Hilbert Spaces*, Princeton University Press, New Jersey, 2005.
10. G. Teschl, *Topics in Real Analysis*, 2020.
 URL: <https://www.mat.univie.ac.at/~gerald/ftp/book-ra/ra.pdf>
11. T. Tao, *An Introduction to Measure Theory*, 2011.
 URL: <https://www.stat.rice.edu/~dobelman/courses/texts/qualify/Measure.Theory.Tao.pdf>
12. M. Taylor, *Measure Theory and Integration*, American Mathematical Society, Providence, Rhode Island, 2006.

I. RUBRICS FOR ASSESSMENT

ORAL PRESENTATIONS

| CRITERIA | Poor 1 | Fair 2 | Satisfactory 3 | Very Good 4 | Excellent 5 |
|---------------------------------|--|---|--|--|---|
| Content Knowledge 50% | Little understanding of the topic | Understands the topic but unable to answer the audience's questions | Familiarity of the topic, answers few or some of the audience's questions | Familiarity of the topic, answers the audience's questions elaborately | Mastery of the topic, discusses minimum key points with the most impact, answers the audience's questions elaborately |
| Delivery 20% | No script, incomprehensible and inaudible voice | Unfamiliarity of script, clear but monotone voice with several mispronunciations of terms | Familiarity of script, clear voice with few mispronunciations of terms | Familiarity of script, clear voice with correct pronunciation of terms | Mastery of script, clear voice with correct pronunciation of terms |
| Visuals 15% | Too much text with no visual aids, unreadable presentation with irrelevant information, several typographical errors | Readable presentation but with little visual aids, several typographical errors | Readable presentation, and uses some visual aids vital to the presentation, few typographical errors | Comprehensive presentation, makes use of appealing visual aids vital to the presentation, no typographical error | Comprehensive presentation, makes use of appealing, effective and varying visual aids vital to the presentation, no typographical error |



| | | | | | |
|----------------------------|---|--|--|--|--|
| Organization 15% | Untimely and unstructured presentation delivery with no logical flow of information | Structured but with confusing flow of information, or untimely presentation delivery | Structured with logical flow of information, or untimely presentation delivery | Well-structured, timely presentation delivery with logical flow of information | Well-structured, timely presentation delivery with logical and interesting flow of information |
|----------------------------|---|--|--|--|--|

Examples of visual aids: Figures, Tables, Graphs, Charts, Videos, etc.

| PROBLEM SETS/EXAMINATIONS | | | | | |
|--|---|--|--|---|---|
| CRITERIA | Poor 1 | Fair 2 | Satisfactory 3 | Very Good 4 | Excellent 5 |
| Problem Interpretation 30% | Minimal or little indication of ideas relevant to the problem | Correct but insufficient interpretation of the problem | Correct but with significantly inaccurate or unnecessary concepts used in the solution | Correct but with slightly inaccurate or unnecessary concepts used in the solution | Correct and accurate interpretation of the problem |
| Proof/Solution Correctness 70% | Incorrect or incoherent solution | Logically sound but incomplete solution | Correct solution but with significantly unnecessary steps or insufficient concepts | Correct solution but with slightly unnecessary steps or insufficient concepts | Well-written, logically correct, and clear solution |



THE UNIVERSITY OF THE PHILIPPINES

UP was founded in 1908 with its first campus in Manila. It was followed soon after by the establishment of constituent universities and campuses all over the country. Over the course of a century, UP has established eight constituent universities distributed across 17 campuses.

Vision

The University of the Philippines (UP) envisions itself to be a leading regional and global university in an environment that sustains 21st-century learning, knowledge, creation, and public service for society and humanity.

Mandates

As the national university, UP is mandated to perform its unique and distinctive leadership in higher education and development, in terms of:

- Setting academic standards and initiating innovation in teaching, research, and faculty development in an environment of academic freedom;
- Serving as a graduate university providing advanced and specialized studies, especially to the faculty members of state and private colleges and universities;
- Serving as a research university in various fields of expertise and contributing to the dissemination and application of new knowledge;
- Leading as a public service university by providing different forms of community, public, and volunteer service to the government, the private sector, and civil society;
- Protecting and promoting the professional and economic right and welfare of its academic and non-academic personnel;
- Providing learning opportunities in various forms to promote such special concerns as responsible citizenship, sustainable development, sports and health development, and cultural development;
- Serving as a hub for regional and global academic networks; and
- Applying the highest standards of academic and institutional governance within a meritocracy based on collegiality, representation, accountability, transparency, and active participation of all constituents.

For more information on the University's Vision and Mission, refer to the [UP Strategic Plan 2017-2023](#).

University of the Philippines' Philosophy of Education and Graduate Attributes

A UP education seeks to produce graduates imbued with an abiding sense of responsibility to their people and nation, the skills and mindsets to improve human life, and a commitment to the freedom and welfare of all.

Aside from mastery of knowledge in their specific disciplines, UP graduates must possess breadth of mind, strength of character, and generosity of spirit, fostered by a firm grounding in both the arts and sciences, and such specialist courses as their programs may require.

They must be prepared to inclusively engage with society and the world at large, mindful of their people's needs and capabilities, and keen to the challenges and opportunities of national development in this century of rapid global change.

UP aims to achieve this through its General Education program, one that develops mind, body and spirit, which familiarize all its students with their culture and history and fosters a sense of shared citizenship, while equipping them with critical thinking, discernment and technical skills they will need to excel in their chosen professions.

THE UNIVERSITY OF THE PHILIPPINES BAGUIO

Established through the initiative of UP alumni in Baguio and Benguet, the University of the Philippines Baguio was inaugurated as a degree-granting unit of the University on 22 April 1961. A land grant worked



out by alumni, the City Council, and by then UP President Vicente Cinco situated the College on its present location, a pine clad-hill offering a scenic view of Baguio. The College went on to make its presence felt as it served as the site of the National Arts Festivals in the coming years. Moves were made to strengthen its research capabilities, culminating in the institution of the Cordillera Studies Center in 1983. Directions towards autonomy began with strategic planning in 1996. The following years saw the College working assiduously in the reformulation and strengthening of its academic programs, primarily. Administration of the College likewise oversaw the development in infrastructure and improvement of services and facilities. Such growth led to the elevation of UP College Baguio to full autonomous status, granted by the Board of Regents in December 2002. UP Baguio is now the seventh constituent university of the UP System.

Vision

As a constituent university of the University of the Philippines System, UP Baguio will sustain its lead position in the delivery of tertiary education in the north. It will continue to nurture and develop innovative programs in the arts and sciences. It will also continue to develop the niche it has created over the past decades in Cordillera Studies.

Mission

Our mission, therefore, as a unit of the U.P. System and as the leading institution of higher learning in Northern Luzon, is to spearhead the offering of the highest standard of education and to contribute to the overall upgrading of the quality of instruction in the region. We seek to create an impact by informing our programs with a regional perspective, at the same time that these are informed by a national and global outlook.

For more information on UP Baguio's Vision and Mission, refer to the [official UP Baguio webpage](#).

THE COLLEGE OF SCIENCE

After the reorganization of UP Baguio during its institution as the seventh constituent university of the UP system in 2002, the College of Science (CS), being one of the three colleges emerging from the reorganization, evolved from the merger of the Division of Natural Sciences and Mathematics and the Sports, Physical Education and Recreation Division. The College offers four undergraduate degree programs: BS Biology, BS Computer Science, BS Mathematics, and BS Physics program. All these programs are regularly reviewed and upgraded to prepare students for careers in education, research, or postgraduate studies.

The CS Dean, together with the Faculty Assembly and in cooperation with the College Executive Board (CEB), leads the Department of Biology, the Department of Physical Sciences, the Department of Mathematics and Computer Science, and the Human Kinetics Program towards academic excellence and public service.

Vision

The College of Science aims to continue offering high standard and relevant quality education through good practices in program implementation that follow innovative pedagogical strategies that utilize appropriate technology in supporting this endeavor. It will further its objectives by initiating interdisciplinary programs anchored on disciplinary specializations in its efforts to enhance the efficiency of research conduct and management. It will continue to encourage and reward scientific productivity by conducting research responsive to the needs of the region, nation, and the global community.

The College envisions itself moving towards a more inclusive and equitable environment that enables faculty members to lead on with exemplary qualifications – mindful of scholarly research and dedicated to public service.

Furthermore, it will support UP Baguio's wellness program for both academic and support staff to guarantee efficiency in service to the university and the society.



Lastly, the College, together with the University, will take initiative to continue, strengthen and widen the reach of its involvement in public service by sharing individual and collective expertise with other academic institutions, local government units, NGOs, peoples' organizations, and indigenous communities in the region and other areas.

Mission

In line with the college's vision, it is our mission, therefore, to produce scientific leaders and civic-minded citizens with high regard for integrity, compassion, and genuine service who lead in a research study that follows ethical standards and excellence in instruction, research, and public service.

It is our mission to improve on basic facilities and design where researchers can work more collaboratively and efficiently. We pursue to guarantee the safety of researchers, to minimize adverse impact to the environment, to respect research protocols involving indigenous communities, and to ensure professional conduct as we encourage good instruction, research, and public service in upholding the University's banner of *Honor and Excellence*.

Goals

In accordance with the mission and vision of the University, the college aims to accomplish the following goals:

- To continue formulating new degree programs while regularly upgrading existing ones;
- To encourage interdisciplinary research across programs;
- To institutionalize the Science Research Center in continuing research responsive to the need of the region, and in the enhancement of interdisciplinary collaboration within the departments of the College and even with other faculty members in other colleges of the University of the Philippines Baguio;
- To have a closer linkage with the Cordillera Studies Center as the university's research center and as an aid in putting up the biodiversity and innovation research center;
- To foster an environment suitable for the growth of the academic and support staff; and,
- To provide public service based on each academic and support staff's specialization.

For more information on CS' Vision, Mission, and Goals, refer to the [official CS website](#).

THE DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

The Department of Mathematics and Computer Science (DMCS) grew from a discipline to a department in 2002 when UP Baguio became the seventh constituent university of the UP System. The Department offers two undergraduate programs (BS Mathematics and BS Computer Science) and two graduate programs (MS Mathematics and PhD Mathematics). The Department pioneered the PhD Mathematics Program in Northern Luzon.

Vision

The DMCS adheres to the highest standards of excellence in all aspects of teaching, research, and extension service. It will build and maintain nationally and internationally recognized experts in the core and emerging areas of study in mathematics, statistics, and computer science. It will set the standards for promoting quality instruction, interdisciplinary research, teacher training, and other extension programs within the University, the Northern Luzon region, and the country.

Further, it aspires to become a Center of Excellence in mathematics both in the country and in the ASEAN region.

Mission and Goals

The DMCS is committed to pursuing excellence in teaching, research, and extension service within the University, the Northern Luzon region, and the country.



The Department identified these five major goals to help realize its mission:

1. Strengthen its graduate and undergraduate programs;
2. Aggressively promote and maintain high standards of quality education;
3. Lead in research capability building and research-generating activities in the mathematical and computing sciences and in mathematics education in the region;
4. Upgrade the quality of mathematics and computing education in the primary, secondary, and tertiary levels in Northern Luzon; and
5. Make quality mathematics and computer science education accessible to the people in the region.

The MS Mathematics Program

The Master of Science in Mathematics (MS Math) Program of UP Baguio, instituted in 2001, provides students with a firm grounding in theoretical Mathematics to prepare them for doctoral studies, research, careers in industry and government, and for teaching junior and senior level undergraduate Mathematics courses.

The MS Math curriculum is designed to allow the student an in-depth study of standard graduate courses in Analysis and Algebra, and electives that would lead the student to a specialization area of Mathematics in preparation for his/her research work. The curriculum also helps the student in applying mathematical tools to problems in other disciplines, such as Engineering, Physics, Life Sciences, Computing Sciences, Statistics, Finance and even the Social Sciences.

For more information on DMCS' Vision, Mission and Goals, and the MS Mathematics Program, refer to the [official DMCS website](#).

Updated as of April 2023